Hop-Constrained Oblivious Routings

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STOC 2021

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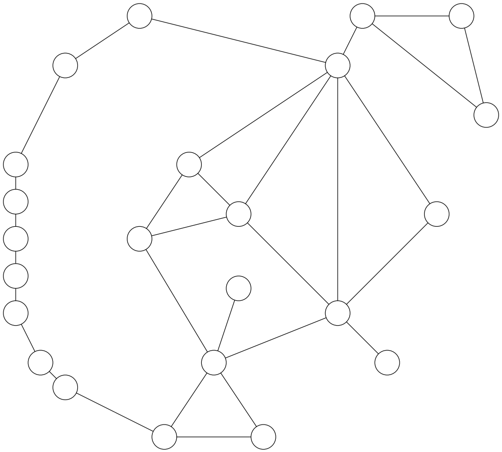
Motivating problem

- Graph G (undirected, unweighted).



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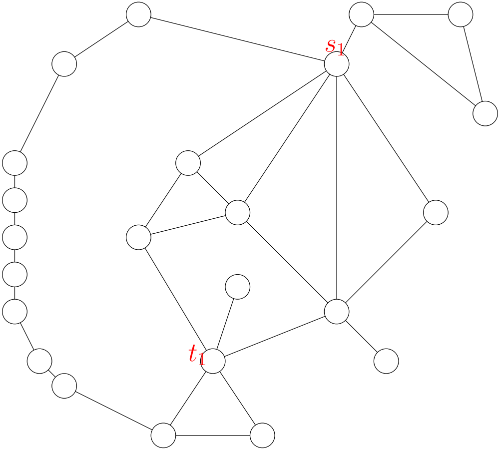

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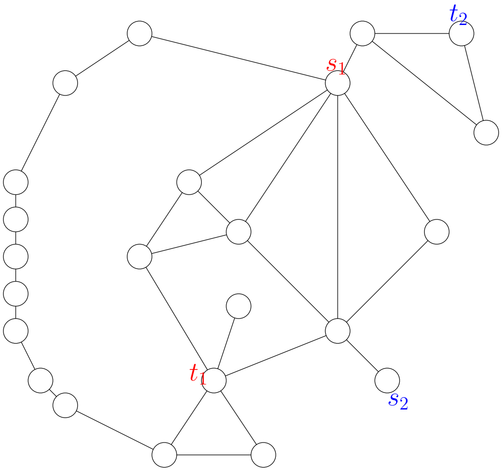

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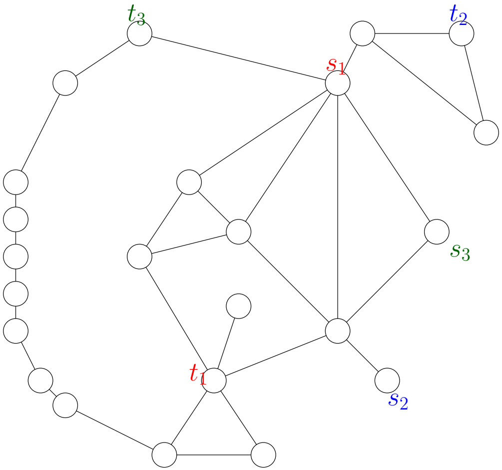

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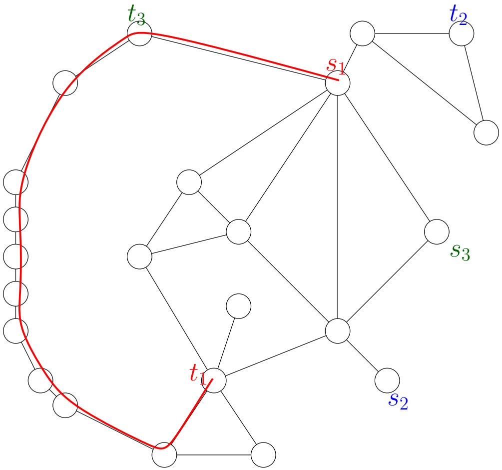

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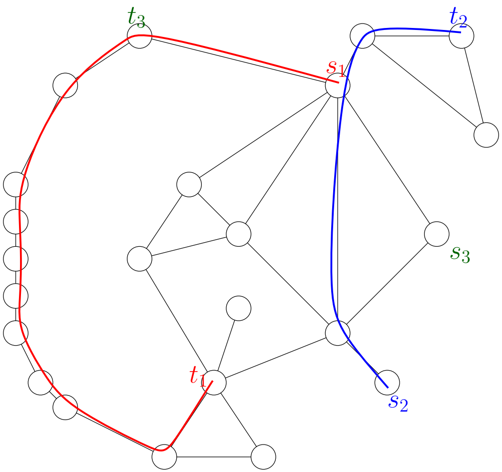

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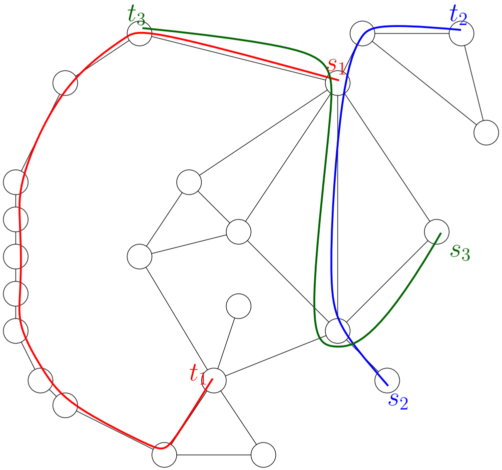

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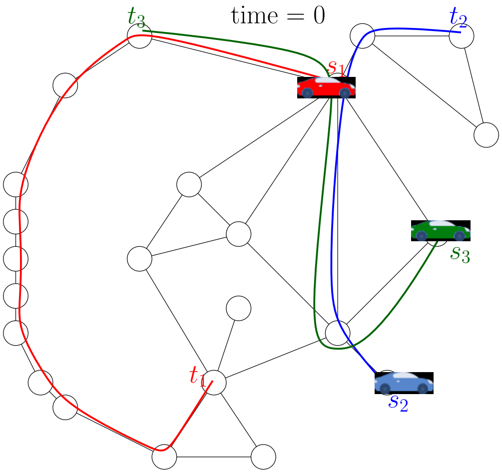

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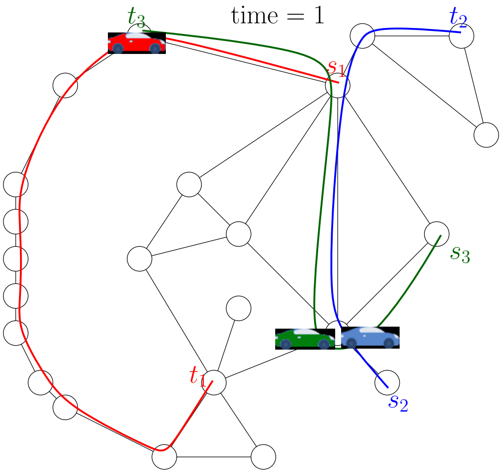

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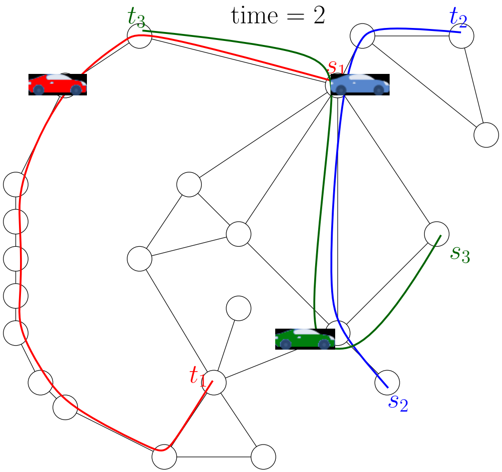

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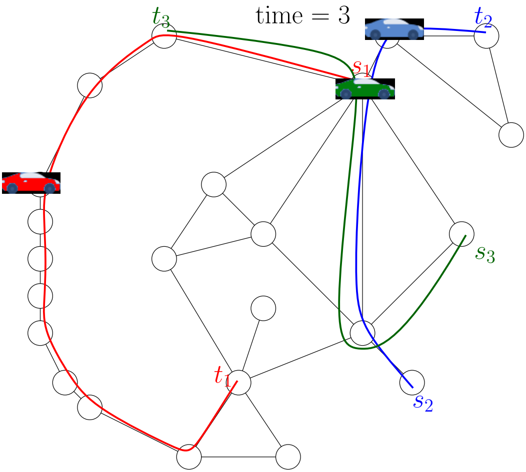

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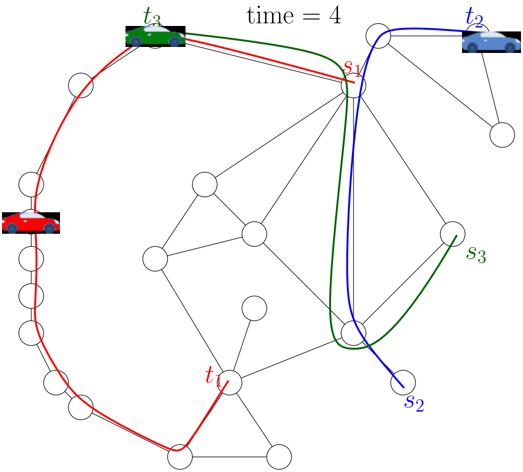

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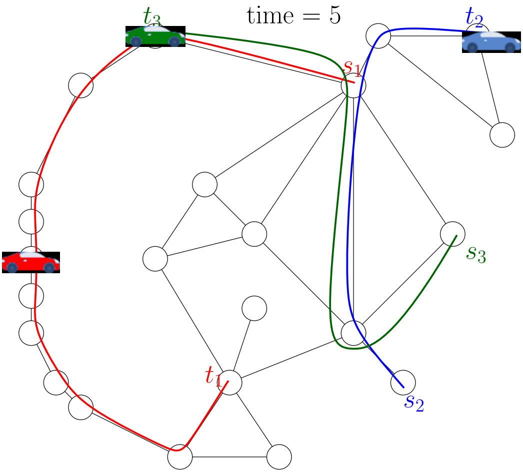

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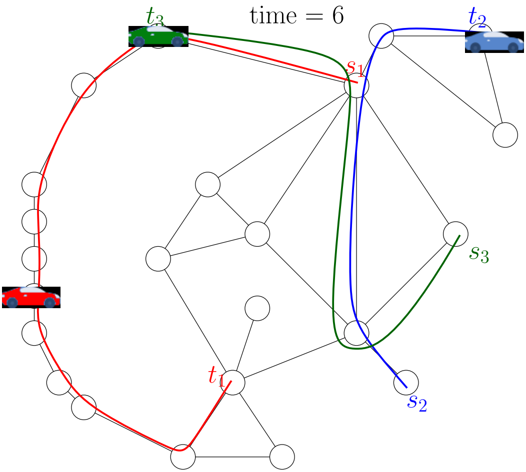

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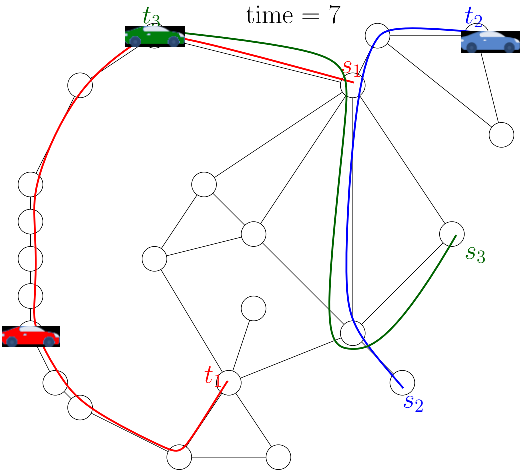

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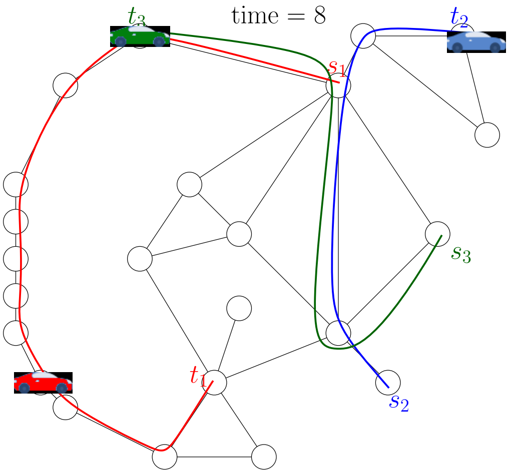

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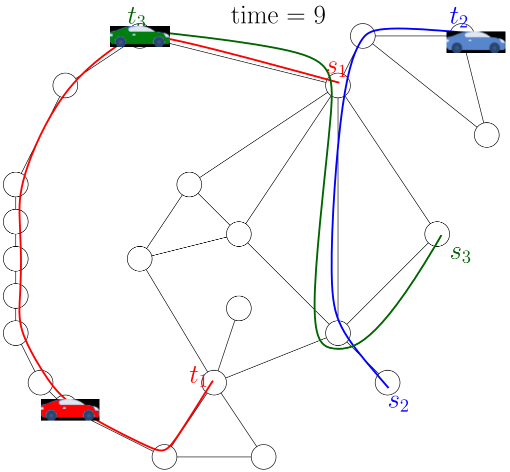

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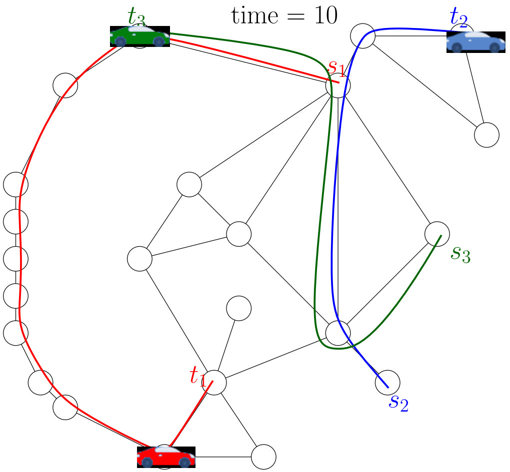

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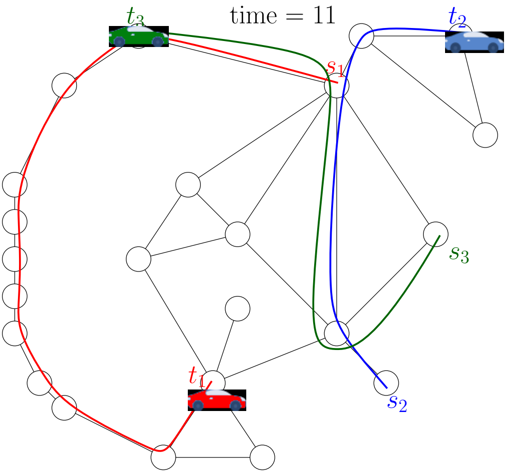

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- Graph G (undirected, unweighted).
- **Input:** source-sink demands.
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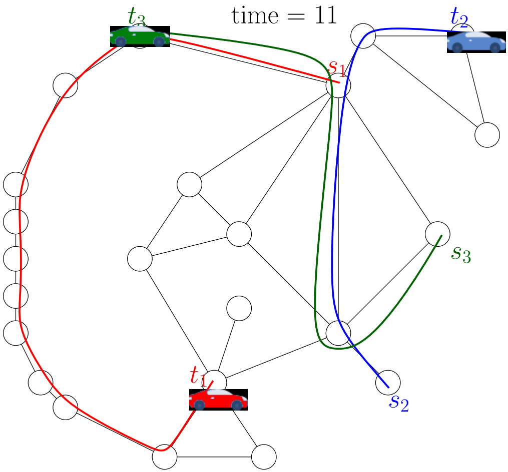
- **Objective:** min. makespan.
- Paths are chosen obliviously.



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Intuition: each driver asks an offline mobile navigation app to produce a path (given a starting point s_i and destination t_i).

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Definition

Given $G = (V, E)$, an **oblivious routing** R is a collection of $|V|^2$ distributions $R = \{R_{u,v}\}_{u,v \in V}$, where for each pair of nodes $u, v \in V$ we have a distribution $R_{u,v}$ of paths between u and v.

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Obliviousnes: All drivers sample from the same R. Note: path chosen by driver i is independent (i.e. **oblivious**) of the path chosen by driver j.

Question—informal

Given G, does there exist a single oblivious routing $R(G)$ whose makespan is $\tilde{O}(1)$-competitive with offline optimum for all demands?

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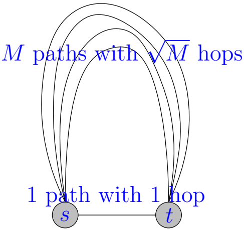
Impossible! $\boxed{\text{No single oblivious routing suffices!}}$ [Räcke, Thesis, '03]



- 1 demand → send along short path. Makespan = 1.
- M demands → send along long paths. Makespan = $\sqrt{M}$.

Overcoming the lower bound

Perfect obliviousness is a very strict constraint. We can get the next best thing!

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Our result—informal

For every graph G and $\mathrm{OPT} > 0$, there exists a **single** oblivious routing $R(G, \mathrm{OPT})$ whose makespan is $\tilde{O}(\mathrm{OPT})$ for all demands whose offline makespan is $\tilde{\Theta}(OPT)$.

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The above (near-) oblivious routing typically good enough.

- Guess OPT.
- Drivers sample a path from $R(G, \mathrm{OPT})$ and drive along it.
- If successful, we are done! Otherwise, double OPT.
- Guessing OPT loses an insignificant $\tilde{O}(1)$ factor.

Oblivious makespan minimization (also called oblivious congestion + dilation or $C + D$ minimization):

Prior work 1/2

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- Hypercubes has $O(\log n)$-competitive makespan-minimizing oblivious routings.
  - "Valiant's trick"
  - Each drivers $s \to t$ picks a uniformly random intermediate m.
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- Similarly, expanders.
- Grids, fat trees, etc.

Prior work 2/2

[Aspnes et al., 2006] titled "Eight open problems in distributed computing":

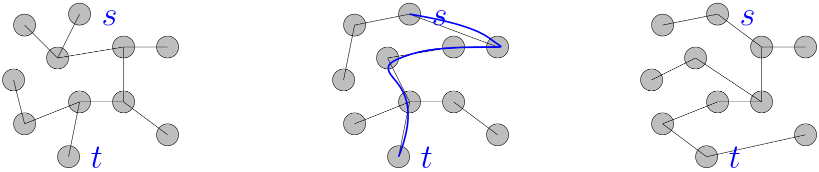
> *Another important open problem is to find classes of networks in which oblivious routing gives C+D [congestion + dilation] close to the off-line optimal... Such a result have immediate consequences in packet scheduling algorithms.*

It seems like our result for all graphs G was missed.

- In spite of being a prominent open problem and special graphs having received considerable attention.
- Probably due to the impossibility result.
- Simply showing the existence is quite technically involved.

Definition (Tree-based routing R)

There is a collection of trees $T_1, \ldots, T_k$. Each demand s, t picks a random tree and routes along it.



Barrier: tree-based routings do not suffice

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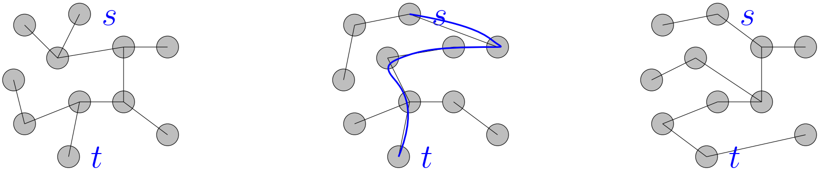
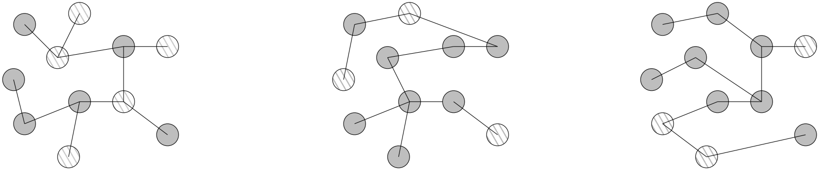


All previously considered constructions of oblivious routings were tree-based.

Barrier

There exists a graph G such that there exists no $\tilde{O}(1)$-competitive tree-based routing.

Partial trees: different trees embed different sets of nodes.



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Solution: Partial tree embeddings

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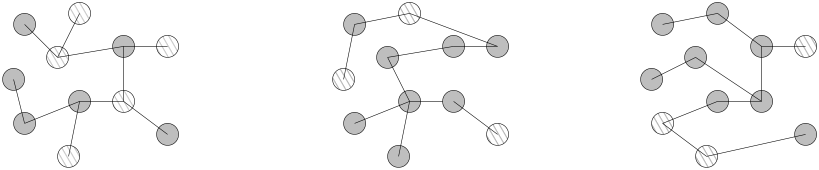

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Theorem

For any graph G and OPT*, there is a distribution over partial tree embeddings such that* 50% *of all demands that can be routed in* $\tilde{O}(\mathrm{OPT})$ *time are routed in* $\tilde{O}(\mathrm{OPT})$ *time.*

Note: if the source s or t are not in tree, this is an "error".
Error correction: one can fully eliminate errors with a complicated scheme described in the paper.

Application: Universally-optimal distributed algorithms
(original motivation)

- Problem: distributed minimum spanning tree, SSSP, min-cut...
- Goal: an algorithm that is as fast as possible for a given network G (up to polylogs).
- We get [HW**Z**, STOC'21]: if the network G is known in advance (but not the input!), there is a single algorithm that is fast as possible on all networks.
- Open question: efficient construction of hop-constrained oblivious routings $\implies$ a single distributed algorithm that is optimal on all networks.
- Connection: Many problems are (up to polylogs) equivalent to simple pairwise communication problems.

Bigger picture: Bi-criteria optimization of congestion and dilation.

Generally very interesting but very hard questions.

Cross-disciplinary. More research, better understanding, and new tools are needed.

First few results of this kind:

Connections with other areas 2/2

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  - General-purpose tree embeddings for problems with hop-constraints.
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